

Written Exam Economics Summer 2018

Labour Economics

May 26, 2018 (10-22)

This exam question consists of 5 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

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Exam cheating is for example if you:

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1 Question 1 - Labor demand

Consider a perfectly competitive economy producing a single good with constant returns to scale. The output Y is produced using capital, K , skilled labor S and unskilled labor U :

$$Y = F(K, S, U) \quad (1)$$

F is twice continuously differentiable and characterized by diminishing returns to each input factor.

1. Write up the optimization problem of a representative entrepreneur in this economy, find the first order conditions and provide a brief interpretation of the first order conditions. [Hint: you may take output as the numeraire.]
2. Total differentiate each first order condition and express the changes in factor prices as a function of changes in factor supplies, income shares and elasticities of substitution, defined as $\sigma_{ij} = \frac{F_{ij}Y}{F_i F_j}$.
3. Suppose capital is skill-neutral meaning that $\sigma_{SK} = \sigma_{UK}$. Show that relative wages do not depend on capital in this economy. What happens to relative wages if the supply of skilled workers increase? [Hint: you may either show it for the general production function in equation (1) or suggest a nested CES production function where relative wages are independent of capital and show it.]

Skilled workers and unskilled workers may not be equally substitutable for capital. Let's consider the following functional form of (1) where $\sigma_{SK} = \sigma_{UK}$ does not hold

$$Y = [A_P P^\gamma + A_N N^\gamma]^{\frac{1}{\gamma}} \quad (2)$$

where

$$\begin{aligned} N &= S \\ P &= (U^\theta + K^\theta)^{\frac{1}{\theta}} \\ \gamma, \theta &< 1 \end{aligned}$$

Final output Y is produced using production intermediates P and non-production intermediates N . Non-production intermediates simply equal skilled workers while production intermediates are combined using a CES aggregate of unskilled workers U and production machinery K .

4. State the conditions under which each skill group is paid its marginal product and derive an expression for the skill premium ($\frac{w_S}{w_U}$).
5. Under which parameter values does capital growth benefit skilled relative to unskilled workers?
6. Discuss whether capital growth could offset the effect of rising education levels on the relative wages of skilled workers in an economy with a production structure as in equation (2).

2 Question 2 - Human capital and labor market frictions

Consider an economy, which consists of many identical workers and many identical firms. All agents are risk-neutral and maximize expected discounted utility. Workers are infinitely-lived and time is continuous. The economy is populated by N workers and the population is growing with the rate n . Young persons are born into unemployment. Consider such a young person who decides which level of general human capital, i , to invest. The cost of investing in i units of general human capital is αi . We assume that it takes no time to upgrade human capital and that human capital makes the worker more productive when being employed, although at a decreasing rate. Letting the productivity be denoted by $y(i)$, this means that $y'(i) > 0$ and $y''(i) < 0$. Since the economy is large, the individual worker's human capital decision has no implication for the number of firms (or vacancies) entering the economy.

While unemployed, workers have a flow utility of z , which is independent of the workers' human capital level. We assume that z does not include unemployment benefits and that $y(0) > z$. As employed a worker earns a wage, which we for a worker with human capital level i will denote by $w(i)$.

We assume a common discount rate, r , for all workers and firms.

1. Suppose that there are no frictions in the economy so the worker observes wage offers from each firm. Derive an equation which characterizes the level of general human capital that the worker will choose.

Assume now that there are frictions in the labor market such that job matching is a time-consuming and costly process. Assume also that the firm fills a vacancy with the rate $m(\theta)$, where θ is the ratio of vacancies (V) over unemployed (U). We assume that $m'(\theta) < 0$. Workers meet vacant jobs at the rate $\theta m(\theta)$. We assume that the value of being unemployed with human capital i , $V_u(i)$, is given by

$$rV_u(i) = z + \theta m(\theta) (V_e(i) - V_u(i))$$

while the value of being employed with human capital i , $V_e(i)$, is given by the following Bellman equation

$$rV_e(i) = w(i) + q(V_u(i) - V_e(i))$$

where q is the exogenous job destruction rate.

Let the value of a filled job with a worker with human capital i be given by $\Pi_e(i)$, while the value of a vacancy is denoted by Π_v . The Bellman equations for a firm are given by

$$r\Pi_e(i) = y(i) - w(i) + q(\Pi_v - \Pi_e(i))$$

and

$$r\Pi_v = -h + \theta m(\theta) (\Pi_e(\bar{i}) - \Pi_v)$$

where h is the flow cost of having a vacancy and \bar{i} is the common value of human capital for all other workers than the worker whose choice of human capital we consider. We assume free-entry in vacancy creation so $\Pi_v = 0$.

Wages are determined by wage bargaining, where the workers' bargaining share is γ while the firms' bargaining share is $1 - \gamma$. The surplus of the match between a firm and a worker with human capital i is given by $S(i) = V_e(i) - V_u(i) + \Pi_e(i) - \Pi_v$ and the wage $w(i)$ is determined such that

$$V_e(i) - V_u(i) = \gamma S(i)$$

and

$$\Pi_e(i) - \Pi_v = (1 - \gamma) S(i)$$

The worker begins his life by investing in general human capital. After this, he starts out being unemployed and searching for a job. Therefore, the worker's maximization problem is given by

$$\max_i V_u(i) - \alpha i$$

2. Derive an equation, which determines the worker's human capital level [Hint: You need to derive an expression of $V_u(i)$ as a function of model parameters only.]
3. Compare the worker's chosen general human capital level in the frictionless case (i.e. the competitive model) to the chosen general human capital level in the model with frictions (i.e. the matching model). Provide an intuitive explanation of any differences in the chosen level of human capital.

In question 2, the worker maximized the value of being unemployed, whereby the firm's value of the match was not taken into account. For the society define the total value of an unmatched worker with human capital i by

$$\begin{aligned} V_s(i) &= \frac{1}{1+r} [z + \theta m(\theta) S(i) + (1 - \theta m(\theta)) V_s(i)] \Leftrightarrow \\ rV_s(i) &= z + \theta m(\theta) (S(i) - V_s(i)) \end{aligned}$$

4. The social planner's maximization problem is to maximize $V_s(i) - \alpha i$ with respect to the human capital investment. Derive an equation characterizing the level of general human capital of the given worker, which would be optimal for the society [Hint: You should keep the number of vacancies constant]. Compare this optimal human capital level to the worker's chosen level in questions 1 and 2. Provide intuition for any differences.
5. Suppose the government gives a subsidy s such that the cost of investing in general human capital faced by the worker is $(1 - s)\alpha i$. The subsidy is financed by lump-sum taxation levied on all workers, employed and unemployed. What is the optimal subsidy level for i) the frictionless case and ii) the model with frictions?

6. Suppose that there are two types of workers, type H workers with high marginal costs of acquiring human capital, α_H , and type L workers with low marginal costs of acquiring human capital, $\alpha_L < \alpha_H$. Suppose that firms can direct their recruitment to each of the two worker types. This means that we can think of the two worker types living on separate islands and that there is free-entry in vacancy creation on both islands. On which of the islands (H or L), will the general human capital level be highest, the wage be highest and the unemployment be highest. [Hint: You do not need to derive the results and an intuitive answer is sufficient.]